HAND-BOOK

TO THE

Public School Arithmetic

FOR

GRADES I and II

TORONTO

THE COPP, CLARK COMPANY, LIMITED

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PREFACE

This Hand-book to the Public School Arithmetic for Grades I and II is intended for the sole use of the teacher. It explains the intention and the use of every exercise in the text-book. To read intelligently any part the teacher must associate it with the exercise it explains.

It is expected that this hand-book, and the text-book which it accompanies, will prove very helpful to the teachers of ungraded schools, as well as to those in charge of Grade I and Grade II. The experienced will find here suitable exercises, carefully graded, with an accompanying discussion of the aim and treatment of each. The inexperienced will find both material and method clearly presented. The teachers of ungraded schools will be relieved of the heavy task of composing exercises day after day, and will thus be enabled to give more attention to *real teaching*. Neither labour nor pains have been spared in the preparation of these works, and it is hoped that they will prove a modern and intelligent presentation of the subject of Arithmetic in our Public Schools.

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INTRODUCTORY.

The pupils' text-book is intended for the *pupil.* There is no part that he cannot read. When worded problems are used, the wording is sufficiently simple for him to understand, while the thoughts expressed are such as a teacher should put before the pupils of these grades.

The Course of Study has been followed very closely. The work for each grade has a distinct place; in no part will the arrangement in the text be a hindrance to the teacher who aims at following closely the Course of Study. In fact every effort has been made to arrange the exercises so as to help the teacher to keep within the limits prescribed. Even within the course for each grade, the work has been arranged in the order that was considered to be the best for the teacher and the pupils to follow.

This hand-book is intended to contain all that is intended for the *teacher.* In it all such matters are discussed very fully. Modes of presenting

new matter to the class, methods of treating the exercises in the pupils' text-book, cautions regarding solutions, discussions of difficult points, and answers to certain problems, are reserved for the teacher's hand-book. This arrangement decreases the size of the pupils' text-book since it eliminates everything that is a hindrance to the pupil. At the same time, it makes it possible to present quite fully everything that the teacher needs in order to comprehend the purpose and the value of every exercise in the text-book. One of the first results of this plan is the possibility of beginning the pupils' book with the very first work that a child does in arithmetic. At this stage, the pupil cannot read much, and so all directions regarding what the child should do are given in the teacher's book.

The whole work aims at being in harmony with modern methods of teaching and recognized principles of education, as well as in accordance with the prescribed Course of Study. It presents the materials and the exercises just as the teacher would arrange them to put before her class. Consequently she is not compelled to select and cull in order to get suitable work for the pupils. Nor is the teacher compelled to amplify every part; under certain conditions she

may prefer to construct additional problems, but the text aims at making this part of the work as light as possible. It does not aim at supplanting true teaching, it aims at following up a modern lesson, with proper exercises.

In Grades I and II, in dealing with the numbers 1 to 20, the various facts (or "combinations" as they are frequently called), in substraction, addition, quotition, multiplication, and partition, are taught. In general, the *concurrent method* of learning the five kinds of facts, is followed. But this plan has not been followed too slavishly; it has been varied wherever advisable. In the early part, when a number is studied, the five kinds of facts are learned concurrently. But the formal expression of them involving the use of all the signs is proceeded with step by step,—each sign being thoroughly mastered before another sign is taught.

HAND-BOOK

TO

PUBLIC SCHOOL ARITHMETIC.

GRADE I.

Thoroughness, from the very beginning is absolutely necessary. Every new step should be taught thoroughly; afterwards, the pupils should do the exercises in the text that are intended to be based upon that lesson. It is necessary that the pupils should actually do these for themselves.

On **page 1** of the pupils' text-book, there are pictures of the numbers one to four. The lesson suggested should aim at teaching the children to recognize these numbers readily.

A supply of suitable objects should be provided. Coloured cubes are commonly used and serve the purpose very well. They are large enough to be handled easily and will stand wherever they are placed; they are sufficiently attractive in colour and form and the children like to handle them. Other objects such as large coloured beans, small apples, buttons and plasticine balls, may be found useful. But the objects used by a pupil for making any one number should be of the same kind. The teacher who used a bell, an ink bottle,

a base-ball and two chalk brushes to make up an objective number five, was not considered a success in primary work.

It is not necessary that a child studying four should have four blocks, four beans, four buttons, four balls, etc., for it is not the objects but the *number* that he is learning. After they have used blocks, they can imagine they are birds, peaches, horses, marbles, or anything else they are fond of.

The class, if small, may be gathered around a table at the front of the room, but if large, the pupils may work at their desks. See that each has a supply of blocks that are all of the same colour.

First, the teacher asks the pupils to arrange their blocks to make a picture, then another and another, until they have arranged them in every way they can. The teacher arranges four blocks in any way she wishes and asks the class to arrange theirs in the same way, then another, and another, etc., being careful to see that every pupil has observed closely. The teacher will have made the arrangement found on page 1 several times no doubt. If the class do not know how many blocks they have, tell them. Show them this picture again and ask the class to

make it, each time the teacher asking for four. This should be repeated until the response is ready and accurate.

Two should then be taught, then *one*, then *three*, every lesson being well drilled and frequently reviewed. In every case, a number when it is taught should be presented as a *whole*,—never in parts, nor by building up.

For beginners, the number four is one of the best to start with; the *four-ness* of four is more impressive than the *two-ness* of two, while the *one-ness* of one object is often less impressive than the colour or the size, or some other property. However, many teachers do start quite successfully with two.

After the pupils know these numbers fairly well, the teacher may give a brief drill in rapid recognition. Many ways will suggest themselves to the teacher. The teacher having made these picture-numbers on cards, four on one, three on one, etc., she lets the class see one just for an instant. They recognize it and tell her what it is. Then they get a glimpse of another and recognize it. This drill may be given frequently until the pupils answer very quickly and quite accurately.

They may now be taught to use the crayon, and to make rings on the blackboard, using a large

free-arm movement. Each is provided with chalk and an eraser. The teacher says, "Make four," and every child makes four large rings neatly arranged. "Erase." "Make two." "Erase." "Make three." "Erase." And so the game goes on. They soon learn to make the rings neatly and a proper space apart to look well. All possible arrangements of three should be taught and drilled on because these will be needed in the pictures to follow. This drill in making picture-numbers will be helpful in fixing the groups in their minds, not only for these numbers but also for others in the course for this grade.

The teacher should devise a number of little interesting schemes for desk-work for the pupils, so as to provide a sufficient amount of drill and review on every lesson that has been taught.

The Facts of 1-4, page 2.—On **page 2** of the pupils' book, we begin the study of the facts of the numbers 1-4. The line across a picture-number suggests placing a ruler or a splint across an object-number to cut off a part of it.

Suppose the class make a four using their blocks. Then by means of a splint, or a ruler, they cut off two blocks, and see that two blocks are left. Let them make several fours, and from

each group cut off two in as many ways as they can; they see that two blocks are always left. Then the teacher asks questions in which two are taken away from four; perhaps she speaks of birds, dolls, horses, apples or marbles. Then the pupils make up such problems. For busy work after the lesson, they might do over again at their desks the work they have done with blocks and splints. They might now perform all these operations with picture-numbers on the blackboard.

We began with the number four, presented as a whole, and we have made an analysis of it. By taking two from four, the pupils obtain this fact in subtraction :—Two from four leaves two. After this analysis we should take the corresponding synthesis; that is the fact (in addition) that two and two make four, when put together to form one group.

The pupils have before them two blocks on one side of the splint, and two on the other side. When they remove the splint they see that two and two make four. They may now do this in as many ways as they like, and whenever they put two and two together they see that they get four. Just as before the teacher and the pupils may now make up little problems in which this fact is involved. After the lesson they may perform

these operations again with blocks, or with coloured pegs, or with picture-numbers on the board.

Next in order would be the analysis of four to find out how many twos in four. This is a fact in quotition (one of the two kinds of division). It may be taught objectively; in general it is better to teach it objectively, although some of the brighter pupils can easily reason it out from the facts previously studied.

Each pupil, using his blocks, makes a four. He puts down a splint to mark it off in twos, and thus he sees that four is two twos. Oral work in problems and busy work with object-numbers at the desks and with picture-numbers on the board would serve to impress what has been taught.

Following this analysis should come the corresponding synthesis:—Two twos make four,—a fact in multiplication. The brighter pupils may get this by reasoning from the previous quotition fact. But generally the objective work is best for the weaker pupils.

The pupil may use the two twos he had found in the previous operation; by removing the splint he sees they make four. The drill, as before, may consist of oral problems and busy

work using object-numbers on the desk and picture-numbers on the blackboard.

Following the natural order the next step would be the partition fact that one-half of four is two. This may be deferred until page 28 is reached. One method is suggested in connection with that page.

The above shows a definite logical order :—Subtraction, addition, quotition, multiplication, partition. No doubt many cases will arise in which it will be desirable to vary the order. If we begin with taking two from four, we tend naturally to follow the above order and take two from four, put two and two together, find how many twos in four, find that two twos make four, and that one-half of four is two. If we begin by taking one from four we get only two facts :—one from four, three and one, since the other three are axiomatic statements and therefore of no real value to beginners. (They are how many ones in four, four times one is four, and one-fourth of four is one.) Starting now with three from four we teach two facts :—Three from four, one and three.

In teaching two, we have only two facts :—one from two, and one and one. In teaching three we have only four, one from three, two and one, two from three, one and two.

When this has been mastered, the picture of five is introduced and taught in the same way. Five objects are given to each pupil at desks. They are asked to arrange them in as many different ways as possible, the teacher commending any orderly arrangement.

The teacher at the board suggests an arrangement which the pupils make with the objects, then another and another, giving as many as she can. Any one of these arrangements is the picture of five, but the easiest and the one which will be most used (see first one on page 3) should be dwelt upon. The pupils make it again and again until they are perfectly familiar with it. Aim at speed, neatness and accuracy. It is advisable to teach one arrangement as the picture and then use some little device to have the pupils note the others (see page 3). For instance, imagine they are boys or girls or birds, etc. The one who should be in the centre has run away to another place. Then when we want to teach the facts of six, it will not be necessary to change the position of an object.

The picture-numbers one to five should now be reviewed. This can be done first at desks with blocks and then at board, all the pupils, or as many as can be accommodated, should be at

the board, provided with chalk and an eraser. "Make 5." "Erase." "Make 3." "Erase." "Make 4." "Erase." "Make 2." "Erase." "Make 3." "Make that say 5." "Erase." "Make 3 another way." "Make that say 5." "Erase." "Make 3 another way." "Make that say 5." "Erase." "Make 4." "Make that say 5." "Erase." "Make 2." "Make that say 5." "Erase." "Make 5." "Erase 2." "Erase." "Make 5." "Erase 1." (Here it is wise always to erase the one in the centre so as to keep the proper group of 4). "Erase." "Make 5." "Erase 4." (Here it is well to have them erase the picture of 4 leaving the one in the centre). "Erase." "Make 5." "Erase 3." "Erase." "Make 5." "Erase a different 3." "Erase." "Make 5." "Erase a different 3." "Erase."

This exercise may be given in very many different ways. Pupils like to imagine these are birds or rabbits, etc., so the teacher makes use of this fact and dictates problems. "Mary had 5 little birds." (Pupils make the picture of 5). It is not necessary to make pictures of birds, for the rings serve the purpose quite as well. "Two flew away." (Pupils erase 2). They will be quite ready to tell you how many are left without asking. This should not be discouraged, but it is not the object of the lesson to get answers to

the questions. It is to make them familiar with performing the operations. "Erase."

"John has 4 marbles." (Pupils make the picture of 4). "He bought one." (Pupils add one to the four). "Erase."

"Jennie has 3 candies, her mother gives her 2." See that they can make the four different arrangements of three and know where to put the two, so that they will have five.

For seat-work they may be given coloured buttons or pegs and asked to make as many pictures of 5 as they can, using one colour only in each picture.

When they are perfectly familiar with this kind of work they are ready for the next step, the facts of 5, indicated on page 3 of the text-book. If the foregoing steps have been followed closely and the work done thoroughly, there will be no difficulty in this. It will be practically a repetition.

Pupils at the desks with 5 blocks, each having 5 of the same colour for making the group, and a ruler or a splint for parting off the desired number of blocks. "Make 5." "Take away 2." "Make 3 and 2; and now put them together." "Make 5." "Take away 1." "Make 4 and 1; and now put them together." "Make 5." "Take

away 3," etc. Make 2 and 3; and now put them together. Make 5; take away 4. Make 1 and 4; and now put them together. Make 5; take away 2; take away another 2. This by way of review might follow. Mary has 5 dolls. (Pupils make picture of 5). She gave away 2. (Pupils take away 2). How many has she left?

They will see that 2 from 5 leaves 3. "Make 3." "Make that say 5." Also present the same fact in a problem:—Mary has 3 apples. (Pupils make picture of 3). Her mother gave her 2 more. (Pupils add two). How many has she? The fact learned is 3 and 2 make 5.

All the facts can be learned in the same way. Continue this, the teacher giving problems, every pupil performing the operation and thus finding results.

An excellent plan for drill. Pupils at the desks with blocks and rulers or splints, one pupil is asked to give all the facts of five showing with his ruler each as he gives them. The class listen and show the same. This may be done either giving the simple facts or making use of problems. In this way the attention of every pupil is held and each one performs the operations many times. As no two pupils will give them in exactly the same way, the class will be alert.

In connection with the above, it is always wise for the teacher to give the class the type of problem she wishes them to give, changing the type frequently.

With the group of five blocks on each pupil's desk, give problems of different kinds and ask for answers, not allowing the pupils to remove the blocks. Ask the pupils to give problems, each one giving the answer to his own problem, without touching the blocks. He may also ask another to give the answer. They are able now to give all the facts *without performing the operations.*

Remove the blocks, or if at the board, erase the picture-numbers and ask the pupils for the same as has been mentioned above.

As the object is to teach *number*, the teacher should, from the very first, get away from the using of objects as soon as possible. The picture-numbers should be used until the pupils have a perfect mental picture. After that, they are not only useless, but harmful, making the work purely mechanical and unintellectual. The pupils must be trained to *think*, and once they have these picture-numbers in mind, they can perform the operation mentally very readily.

Page 4.—Pupils are given a sheet of paper and ruler and pencil and asked to rule and make picture-numbers as indicated, filling the sheet. In this, it is better to have them make circles only and not fill in making a black dot like those in the book. This will train them in the free oval movement, as well as spacing and neat arrangement of pictures. For this exercise, buttons, plasticene balls, split peas, or coloured pegs may be used.

Page 5.—This is a page of review work. Class may be given blocks and rulers or splints and asked to perform the operations indicated, making up problems about each. Plasticene balls, split peas, coloured pegs or buttons may be used for this, also.

Page 6.—Here the number six is presented and taught in the same way as was the number five, and should be taught *thoroughly* before the class is asked to do the work indicated on page 6 of the text-book.

After the pupils can recognize readily the group of six blocks on the desk, and the picture-number six on the blackboard and can make them easily, the facts of six should be taught. The methods used in teaching the facts of five will be quite as useful in teaching the facts of

six. The following order will be found helpful. Any re-arrangement of the paragraphs (*a*), (*b*), (*c*), etc., but the order of the facts within each paragraph is believed by many teachers to be logical and natural.

(*a*) Six, take away two? Four and two put together? Four is how many twos? Six is how many twos? Three twos make?

(*b*) Six, take away four? Two and four put together?

(*c*) Six take away three? Three and three put together? Six is how many threes? Two threes make?

(*d*) Six, take away one? Five and one put together?

(*e*) Six, take away five? One and five put together?

Page 7 is an exercise similar to that on page 4, using the numbers one to six.

Page 8 is a review exercise, showing how many 2's in 6, 2's in 4, 2's in 5, 2's in 3, 3's in 6, 3's in 5, 3's in 4, 4's in 6, 4's in 5.

The work thus far should not be hurried, plenty of time should be given and each step of

the work done thoroughly. Speed and accuracy are essential to success.

The pupils will now have a thorough knowledge of the numbers one to six, and will not hesitate in answering any question that may be asked.

Page 9.—The figures are now introduced. A new figure is associated with its picture-number. The teacher makes the picture of 4 (say) on the board and then makes the figure 4 near it. To learn its form the teacher and the pupils write it in the air,—all facing the board. Then the teacher gives to each a sheet of paper on which she has made the picture-number and the figure beside it,—both being made large and with a coarse pencil. The pupils make the object-number with plasticene balls, and overlay the figure with plasticene strings, or buttons, or coloured pegs. Later the children make both the object-number and the figure in the same way, but without the sheet.

They now know the form so well that they can easily be taught to write the figure. All the pupils stand at the board with their chalk crayons, and the teacher has them write in the air, making large free-arm movements. Then they make the figure 4 in the air, and afterwards on the black-

board, retaining the large free-arm movement they used in writing in the air.

Every care should be taken to prevent any finger-movement. Writing at the desks should not be begun until sufficient has been done at the board to ensure a proper movement being used.

The figures 1 and 4 may be taught in one lesson; 6 also is easy. But 3, 5, 2 are more difficult; one of them is enough new material for one lesson.

An exercise similar to the one on the bottom of **page 9** should be placed on the board for recognition of figures. A test may be made in any way the teacher may think of. Each pupil may be asked to read a line across or up or down.

A figure may be written by teacher, a pupil asked to make the corresponding picture. The teacher may make a picture and ask for the corresponding figure to be made.

Page 10.—This is an exercise similar to that on page 7, except that the figures are placed beside the picture-number.

Page 11.—As the class know all the facts about the numbers one to six and they are perfectly familiar with the figures, they are now ready for the signs. The plus sign is introduced

here. If it is taught to mean that things are *put together* there will no difficulty arise.

The teacher holds up one block in one hand and one in the other hand. She puts them together and tells the class what she has done, viz., "I have put together 1 and 1." She does this again and again using different numbers. She also asks the class to tell her what she has done in each case. The teacher may now dictate and class do the work, in each case the pupils stating what they did, *e.g.*, "I put together 2 and 1." "I put together 3 and 2." Now show them how the board will say "put together." Teacher shows them the sign, telling them it is made by putting two lines together, the one crossing the other, both perfectly straight.

She may now write on board $4+4$ and ask class what the board tells us to do, pupil answers, "Put together 4 and 4." Continue this until they can tell you promptly. This will form a further drill on the recognition of the figures, as well as the *use* of the sign. With the class at the board, the teacher may dictate class writing. "Put together 2 and 2." "Put together 2 and 1." "Put together 3 and 4," etc.

An exercise similar to one on page 11 may be put on the board and individual pupils may be

asked to read the whole. Now they are ready to read page 11. Notice that no answers are required. It is simply an exercise in reading, testing the class in the recognition of the figures and the *use* of the sign. They should read it in the way indicated above. Do not be satisfied with one reading of this page.

Page 12.—The class is now ready to put these together and give results.

Here these exercises should be made real live problems for the class, not a mere mass of figures.

Teacher says, "Mary has 2 apples (writes 2 on board), her sister gave her 2, (put down 2 a little way from the 2). She put them together (put sign between). How many has she now?" (Write = 4). Continue with a great many examples. It is necessary to present these facts in all possible different ways. (See pages 12 and 13).

"Jennie has 2 apples. Her mother gave her some. She put them together. Now she has 6. (Teacher writes as she speaks 2 + = 6). How many did her mother give her?"

"Jennie had some marbles. She bought 3. Now she has 6. How many did she have at first?" (Teacher writes as she speaks, + 3 = 6).

This method is followed in introducing the different styles.

Just as soon as the pupils thoroughly understand the meaning of the sign "+," give the name "*plus*" and have them always call it "*plus*" (not "and") when reading it.

For practice in using this word, page 11 may be read again, thus "five plus six," "three plus four," etc.

Page 12 may also be read requiring no answers at first, then re-read giving the answers. Thus "one plus one *equals* two," "two plus two *equals* four." Teach the word "equals" and call it "equals," not "are" or "make."

If the previous work has been done thoroughly, there will be no difficulty with pages 12 and 13. These exercises should be read a very great many times until the pupils can read them without any hesitation whatever. Similar exercises may be put on the board for the class to read. It is well to have one pupil read one whole set, *i.e.*, the six examples in one block at one time. If they cannot read them almost as fast as the teacher, they do not know them sufficiently well to receive anything new.

Page 14.—The number seven is presented and taught the same as the preceding numbers.

As the class can see all the pictures in seven, the teacher will find she has very little to teach. It will be simply a matter of guiding and drill. The class will be able to give all the facts of seven the first time the picture is presented to them :

$7 - 4 =$	$7 - 2 =$
$3 + 4 =$	$5 + 2 =$
$7 - 3 =$	$7 - 2 - 2 =$
$4 + 3 =$	$7 =$ how many twos ?
$7 - 3 - 3 =$	$7 - 1 =$
$7 =$ how many threes ?	$6 + 1 =$
$7 - 5 =$	$7 - 6 =$
$2 + 5 =$	$1 + 6 =$

When the pupils know these facts so that they can think them out without seeing the objects or the picture-number, the *figure* 7 should be taught.

When they can make the figure well on the board, they are ready for **page 15.** Note the variety of exercises. The class will readily see that it is the very same fact written in other forms. They should be made familiar with these from the first, then they will not interpret the exercises wrongly, and when they see a plus sign add whatever happens to be placed near it.

Four plus three equals seven, and we can place the seven first or last as we please, thus $4 + 3 = 7$

or $7 = 4 + 3$. (In the second form write $4 + 3$ first and then put the equals = to the left, and seven to the left of that again). The exercises should be read always beginning at the left. If that figure is the one omitted it will necessitate knowing the answer before beginning to read. But this should always be insisted upon, then there will be no hesitation. It is like being able to read a sentence before attempting to read it aloud.

As mentioned before, it is well to have one pupil read a whole set of examples.

Page 16 is for seat-work only. At the top of page 17 there is an exercise that may be used for seat-work. It may be done in plasticene or split peas as well as with pencil. When the pencil is used, there should be a large free-arm movement.

Pages 17 and 18 are exercises of review work on the numbers 1 to 7 inclusive. This will present more difficulty than when the exercises are all confined to facts of one number. Therefore much time should be spent on these and similar exercises.

Interpret these exercises by the use of problems and ask the class to do so, *e.g.*, on page 17.

John has 3 marbles. He buys 2 more. Now he has 5.

Tom has 5 balls. He found 1. Now he has six.

Page 19.—The minus sign may now be introduced. First teach the meaning, using problems. At first let the work be concrete.

As they know all the facts of the numbers 1 to 7 now, it will not be necessary to do much concrete work. Teach that the sign tells us to "take away." Now let the blackboard speak to them. Teacher says, "Mary has 7 apples and she gives away 3. (Writing as she speaks 7 – 3). How many has she left?" 7 – 3 = 4.

The teacher will find it much easier to begin with the larger numbers in teaching the signs.

"Mary has 7 apples. She gives away some. She has 2 left. How many did she give away?" (Writing as she speaks 7 –   = 2).

The class will read this "7 take away 5 leaves 2." Ask pupils to read similar exercises to those on the top of page 19, which are written on the board, not asking for results. The object is to make them familiar with what the sign *means*. Have it read thus, "Take away 1 from 7," "Take away 5 from 7."

Later ask for results.

As soon as they understand the meaning of the sign, give the name "minus."

Have these exercises re-read, now, thus, "7 minus 1," "7 minus 5," "7 minus 4," until they are familiar with the word. Now have them read again giving answers, thus, "7 minus 1 equals 6."

These exercises should be interpreted by the pupils in the form of problems. By doing this constantly at the beginning the work of writing out problems later will be simplified.

There should be great variety in the problems, as the minus sign may be interpreted in so many ways.

A boy had 7 oranges. He ate 1. He had 6 oranges left.

A boy had 7 sheep. 3 died. He had 4 sheep left.

A boy had 7 marbles. He sold 2. He had 5 marbles left.

A boy had 7 balls. He gave away 2. He had 5 balls left.

A boy had 7 books. 2 were burned. He had 5 books left.

Page 20.—This is a review of all the numbers 1 to 7 and the signs plus and minus.

The teacher may dictate problems now which will involve the use of either sign and the class write them on the board. This should be continued until the class do not make a mistake in the use of these signs.

A good test exercise is this. Place figures on board using no signs. Ask the class to put in the correct signs, thus:

2 4 6, 7 3 4,
4 3 1, 6 2 4.

It will be seen that some exercises might be written in two ways, *e.g.*, 4 3 1 may be written $4 = 3 + 1$ or $4 - 3 = 1$.

With **page 21** the print is introduced. Pages 21 and 22 are review exercises using the print.

Page 23.—Present the picture-number *eight*; have it taught and impressed as a whole, just as the preceding numbers were treated. Then review the picture-numbers *one* to *eight*. At this stage the use of objects may be discontinued with classes of average ability. Instead of these, use picture-numbers; they are less concrete and therefore better since the child has by this time made considerable progress "from the concrete to the abstract."

All of the subtraction, addition, quotition, and multiplication facts of eight may now be taught;

the partition facts may either be taught now or deferred until page 33 is reached; the latter is considered by some to be the easier way.

8 − 4 =	8 − 2 =
4 + 4 =	6 + 2 =
8 = how many fours?	8 = how many twos?
2 fours =	4 twos =
8 − 1 =	8 − 7 =
7 + 1 =	1 + 7 =
8 − 3 =	8 − 5 =
5 + 3 =	3 + 5 =
8 − 6 =	
2 + 6 =	

When they know these facts so well that they do not need the aid of the picture-number, teach the *figure* 8. As this is a difficult figure to make, it would be wise to follow pretty fully the method used in teaching 4:—tracing it with plasticene, making it with plasticene, writing it in the air, writing it on the blackboard. A good deal of practice on the blackboard will be needed, and care must, as before, be exercised to make sure that a good, free-arm movement is employed throughout.

As they know all the figures from 1 to 8, they can now write without assistance all the subtraction facts and all the addition facts of eight.

Class at the board. Teacher asks them to write all the addition facts, beginning with the figure 8, thus $8 = 2 + 6$, $8 = 5 + 3$, etc., writing one under the other neatly. Insist on large figures. They may now be asked to write the same putting the 8 on the right hand side, thus $3 + 5 = 8$, $4 + 4 = 8$. Again, write the subtraction facts beginning with 8, thus $8 - 2 = 6$. Again, the subtraction facts putting 8 in the centre, thus $6 = 8 - 2$, etc.

Pages 25, 26, 27 are review.

On **page 28,** a method of teaching one-half is indicated.

As it is difficult for each pupil to have an object as indicated in the text-book, a strip of paper may be given to each, the strips not necessarily the same size.

TEACHER.—Fold your paper so that the two ends will be exactly together. Crease. Open.

TEACHER.—What have you done?

PUPIL.—We have divided our paper into two equal parts.

TEACHER.—We call one part one-half.

TEACHER.—Show me one-half of your paper.

TEACHER.—How did you get one-half?

PUPIL.—We divided it into two equal parts.

TEACHER.—What do we call each part?

PUPIL.—One-half.

The teacher may, by using objects, perform the operation as pupils dictate, to obtain one-half.

Page 29.—Pupils with blocks at desks or at the board may be asked to make the pictures of 8, 6, 4, 2, and asked to divide them into two equal parts as they did the strips of paper. They will readily see the facts and be able to express them as indicated on page 29.

Pages 30 and 31 are review.

On **page 32** a method of teaching one-fourth is indicated.

Following up the method outlined in teaching one-half, the following might be suggested.

Give each pupil a strip of paper.

TEACHER.—Fold it so that the two ends will be exactly together and crease.

TEACHER.—Open.

TEACHER.—What have we done?

PUPIL.—We have divided it into 2 equal parts.

TEACHER.—What do we call each part?

PUPIL.—One-half.

TEACHER.—How many strips of paper had you?

PUPIL.—One.

TEACHER.—How many halves in one?

PUPIL.—Two.

TEACHER.—Fold one end in to meet the centre and crease. Fold the other end to meet the centre and crease. Open.

What have you done with your strip of paper?

PUPIL.—We have divided it into four equal parts.

TEACHER.—We call each part one-fourth.

TEACHER.—How did you get one-fourth?

PUPIL.—We divided it into two equal parts first and then divided each half into two equal parts.

TEACHER.—How many strips of paper had you?

PUPIL.—One.

TEACHER.—How many fourths in one?

PUPIL.—Four.

TEACHER.—Show me one-half of your strip. How many fourths in one-half?

TEACHER.—What is one-fourth?

PUPIL.—One-fourth is one-half of one-half.

TEACHER.—If you tear off one-fourth, how much have you left?

PUPIL.—Three-fourths.

TEACHER.—Which is larger one-half or one-fourth? One-half or three-fourths? One-half or two-fourths?

TEACHER.—Fold your paper so that I can see only three-fourths. If you tear off one-half, how much will you have left?

TEACHER.—Make the picture of eight. Divide it into two equal parts.

TEACHER.—What do we call each part?

PUPIL.—One-half.

TEACHER.—What is one-half of 8.

PUPIL.—Four.

TEACHER.—How do we get one-fourth of anything?

PUPIL.—First divide it into two equal parts and then divide each half into two equal parts.

TEACHER.—What have we done with eight?

PUPIL.—We have divided it into two equal parts.

TEACHER.—To get one-fourth, what shall we do now?

PUPIL.—Divide each half into two equal parts.

TEACHER.—What have you done with 8?

PUPIL.—We have divided it into four equal parts.

TEACHER.—How many are in each part?

PUPIL.—Two.

TEACHER.—What part of eight is two?

PUPIL.—Two is one-fourth of 8.

TEACHER.—What is one-fourth of 8?

PUPIL.—One-fourth of 8 is two.

In the same way one-fourth of four can be taught.

This may be followed up by giving problems.

"Jennie had 8 oranges. She ate one-fourth of them. How many did she eat? How many has she left?"

"Mary had 8 apples. She gave Tom one-fourth of them and Fred one-fourth. How many did she give away? How many had she left?"

On **page 34** columns are introduced for adding. Here, the class should not say 4 and 2 make 6, 5 and 3 make 8, etc. Teach them from the first to add columns properly, giving results only.

On **page 35** the picture of 9 is presented. This and all the facts of 9 should be thoroughly learned;

9 − 5 =	7 + 2 =
4 + 5 =	9 = how many twos?
9 − 1 =	9 − 6 =
8 + 1 =	3 + 6 =
9 − 8 =	9 − 3 =
1 + 8 =	6 + 3 =
9 − 4 =	9 = how many threes?
5 + 4 =	3 threes =
9 = how many fours?	9 − 7 =
9 − 2 =	2 + 7 =

Teach the figure.

Page 36 contains exercises on 9 only. Here teach 0; as in the preceding get the idea first and the symbol afterwards. Pages 37, 38 and 39 are review work on the numbers 1 to 9 inclusive.

On **page 40** one-third is introduced. This may be taught by using a similar method to that outlined for teaching one-fourth. Note the arrangement of figures on page 41.

Page 43.—To introduce this style of work it is well to use problems. "Mary had 9 apples.

She gives Jennie 1 and Alice 1. How many did she give away? How many has she left?"

The teacher gives several problems, writing them (in figures and signs) on the board as she speaks. Continue until class understand. Class give problems and teacher write on board. Class give problems, writing them on board themselves.

They *should not be told* that 1 and 1 are added together. This work should be continued until they see that is what can be done. Then these and similar exercises should be read:—9 minus 2 equals 7; 9 minus 5 equals 4; 9 minus 6 equals 3. While this is not the only way that that exercise can be read, it is the simplest. If a pupil reads it aloud as it is, viz., 9 minus 1 minus 1, his whole attention must necessarily be placed on the reading. It is always wise to simplify the reading as much as possible.

Such exercises as these $9 = 2 + 3 + \quad$, should be read, 9 equals 5 plus 4 and $3 + 5 + \quad = 9$, should be read 8 plus 1 equals 9. This simplifies the work and thus two operations are performed with the one reading.

The teacher will find that interpreting these exercises on pages 43 and 44 and 46 in the form of problems will be an invaluable exercise.

Pages 47 and 48.—The picture of 10 is taught, also all the facts.

$10 - 5 =$
$5 + 5 =$
$10 =$ how many fives ?
2 fives $=$
$10 - 3 =$
$7 + 3 =$
$10 =$ how many threes ?
$10 - 7 =$
$3 + 7 =$
$10 - 1 =$
$9 + 1 =$
$10 - 9 =$

$1 + 9 =$
$10 - 2 =$
$8 + 2 =$
$10 =$ how many twos ?
5 twos $=$
$10 - 4 =$
$6 + 4 =$
$10 =$ how many fours ?
$10 - 6 =$
$4 + 6 =$
$10 - 10 =$

When the class know these they are ready to learn how to write ten. This may be done by the use of splints. Tie ten in one bundle. Make several bundles of ten. Tell the class that we have no more figures to learn. We must find some other way to write ten. The teacher, holding up one bundle, leads the class to see that there is only one bundle, although there are ten splints in it. Hold up two bundles, three, four, five, etc., and let the class become familiar with this new idea of one, two, three.

The teacher must tell the class how to write the ten. It is easier to present it if some single

ones are shown with the tens, *e.g.*:—Teacher holds up 1 bundle and 2 single ones, and shows class that the 1 is placed on the left side and the 2 beside it.

Similar examples using any number of tens and any number of single ones should be given, until the class can write them very readily.

Then holding up tens only, the class will see there are no single ones and the nought is written beside the number of tens. Continue this until the class can do it very readily and can interpret anything written by the teacher, *i.e.*, any number containing tens and units only.

Page 49 contains exercises on the number 10.

Page 50 has simple exercises for review.

The exercise on the bottom of page 52 should be read, 10 minus 4 equals 6, 10 minus 3 equals 7, and so on.

Pages 53, 55, 56 are similar to pages 43 and 44.

These will present no difficulty for the pupils will be able to read them at sight without any hesitation.

On **page 54,** subtraction is introduced in another form. Here the class may give the answer only. If they do read it, they should say 4 from 10 leaves 6, 3 from 9 leaves 6, etc.

Pages 55 and 56 are review exercises. An exercise similar to the first one on page 55, may be read in different ways.

First, the work may be done mentally, the pupils giving results only, *e.g.*, 3, 7, 9; 5, 7, 8.

Second, it may be read adding the first two together, thus 7 plus 2 equals 9.

Third, it may be read, adding the second and third figures, thus 3 plus 6 equals 9. By using a variety of methods each exercise affords added drill.

Page 57 of the text-book presents problems in which the word "times" is used.

This may be introduced by means of a few oral questions. 8 minus 4 equals? 4 plus how many, equals eight? 8 is how many fours? If there are eight apples in a basket, how many times could I put in my hand and take out four? A little girl can hold 4 candies in her hand, how many times should she take four, if she is only to have 8?

I went to your stable twice, and each time I took away four horses, how many of your horses did I take away? How many times did I take four? Two times 4 equals how many?

$10 - 2 =$. $8 + 2 =$. 8 equals how many twos? How many twos make 10? If

you are to take 10 plums how many times should you take two? How many times 2 equals ten?

All of **pages 57** and **58** consist of problems of this nature, presented in three different ways.

On **page 59** the letter "t" is used for "times." The teacher may ask the pupils the first sound in the word "times." After they have given it she may write the letter "t" on the board, and tell them that they are going to use that to mean "times." This symbol is used frequently on pages 59 and 60, and so by that time they will be able to use it freely and intelligently.

Before reading the exercises on the bottom of page 60, the pupils should know, and be able to read without any hesitation, the preceding exercises using the symbol "t." A good plan is to have them give results only, then when they come to read the exercise mentioned above, they will be able to read it thus,—"ten minus six equals four," "eight minus six equals two," "nine minus four equals five" and so on.

Page 63.—Since the pupils know that 3 t 3 = 9 and 2 t 4 = 8, they can read 3 t 3 − 2 t 4 thus "nine minus eight equals one."

Again, 4 t 2 = 8, and 3 t 1 = 3, then 4 t 2 − 3 t 1 should be read "eight minus three equals five."

The use of the letter "t" as a sign meaning "times" is meeting with favour everywhere. It has been introduced by several teachers living at widely distant places throughout the country. Generally a teacher, who has begun its use quite independently, is greatly surprised when she learns that several other teachers are using it. Teachers, to whom it is new, like it. The pupils already know the sound of the letter "t," and as this is the initial sound in the word "times" they readily see that you are using it for that word. It is this appropriateness of the symbol, that make it so easy for the child to learn it, and to remember it.

The use of the "×" sign is still quite common. Unfortunately some teach it with one meaning, some with another. Many say it means "times"; many others teach that it means "multiplied by."

A large number of primary teachers prefer the former interpretation,—many of them admitting that it is not strictly correct. Others contend that usage *now* justifies it, and that usage, and usage alone,—is the supreme arbiter of the meaning of any symbol or word in the language.

Many other teachers contend that "×" was formerly "multiplied by," and that this meaning never was changed. They claim that even those

who use the other interpretation frequently read the sign "×" as "multiplied by," especially in such problems in formal multiplication as 4836 × 2759. They claim that the usage referred to may be far less wide-spread than has been claimed; and that it is not used by any authors,—if we have any such,—who are recognized as authorities in such matters; in the choice and use of words the usage of the best writers must be considered, but such authors can hardly be quoted in this case.

Again, in more advanced mathematics the expression "multiplied by" is needed and cannot be avoided; some symbol must therefore be retained to mean this expression, and so the sign "×" is likely in advanced work to retain this meaning. And if this is the correct interpretation in advanced work, it should be so regarded in elementary work.

It is admitted that the one sign should not bear two interpretations. The present confusion is causing pupils to say "five cents times three," and "three multiplied by five cents." Such blunders are quite indefensible. They cannot be excused on the ground that the number of cents (in this illustration) is "three times five" and that this would give the same number as "five times three." It is plain to the teachers of all

the junior grades that all confusion must be avoided. We must, therefore, have one symbol that means "times" and another that means "multiplied by." No one objects to the use of the letter "t" as a symbol for the former, nor to the use of the cross "×" as a symbol for the latter.

On **page 64** of the text-book, there are several little problems in quotition,—*how many* fives? *How many* fours? *How many* threes? In some schools, quotition problems are called the "how many" problems.

After a few oral problems, the teacher may express one of them thus,—"8 is how many fours?" A pupil answers "8 is 2 fours." This the teacher writes down thus,—8 = 2 (4), and tells them to read it thus,—"8 equals 2 fours." They should now practise reading some of these from the text-book:—"6 equals how many threes?" "8 equals how many fours?" After the pupils have become accustomed to reading these questions correctly, they might answer them thus,—"6 = 2 threes." They should also have practice in putting down their answers thus,—6 = 2 (3); 8 = 2 (4); 9 = 3 (3); 10 = 2 (5).

The last part of page 64 introduces a new sign, the division sign. Of the meaning of this sign there are two distinct explanations, and as both of these have, for years, been accepted as

correct, it is quite impossible to reject either of them as incorrect.

This problem,—"How many apples at 2 cents each can I buy with 10 cents?"—is distinctly a problem in *quotition.* It asks how many twos there are in ten,—or how many times can 2 cents be taken from 10 cents. This is one kind of division problem. Therefore the expression "divide" or "divided by" admits of this interpretation, and so some teachers may quite fairly take the problem "10 ÷ 2" (or "ten divided by two") to mean "how many twos are there in ten?"

But instead of being a problem in quotition "10 ÷ 2" may be a problem in *partition.* In this example,—"I paid 10 cents for 2 oranges, what did each orange cost?" the result is "one-half of 10 cents." It is quite correct to express this using the division sign 10 c. ÷ 2,—meaning that the 10 cents must be *divided* into 2 equal parts.

What then should the teacher do? To confuse *quotition* problems with problems in *partition,* would not be pardonable. Some teachers solve the difficulty by expressing quotition thus: "10 = (2)," "ten equals how many twos?"—while they express the partition problem thus "one-half of ten is five." This enables them to

omit entirely the use of the division sign until the end of the Grade III. course, where it can easily be handled in a satisfactory manner. This solution is so satisfactory that no teacher who prefers this plan should be asked to abandon it; she would do better to have her classes omit all problems in which the division sign occurs.

A large number of teachers *use the division sign to mean quotition only.* For these the following suggestions will be found helpful:

First give the class a sufficient number of carefully worded quotition problems. If plums cost 3 cents each, how many could you buy with 9 cents? How many stalls would be needed for 10 horses if each stall holds 2 horses? The class will see that they must find out how many threes there are in nine, and how many twos there are in ten. The teacher may now introduce the new expression of this problem, thus $9 \div 3$, $10 \div 2$,—being sure that they have the thought clearly in mind. The pupils should now make up problems to fit other expressions, $8 \div 2$, $10 \div 5$, $6 \div 3$, and so on. Then the teacher should tell them how to read these:—"8 divided by 2" meaning "eight broken up into twos," "10 divided by 5" meaning "ten broken up into fives,"—and so on. For practice they should now read several of these expressions, the teacher being careful to

see that they know what each one means and that they use the correct expression "divided by" in every case. They may now answer all these problems thus,—"ten divided by two equals five."

There are no doubt several teachers *who take the division sign to mean partition only.* In introducing this new sign they should use several well-worded partition problems, then introduce the sign. They will probably say $10 \div 2$ suggests "ten divided into two parts (or "ten broken into two parts"). Then the class should read several columns of these exercises, thus "ten divided by two." Afterwards they should give the answers, thus,—"ten divided by two equals five." Many of the suggestions regarding accuracy, thoroughness, and practice will be found quite applicable here also.

On **page 65** there are exercises in which the minus or plus sign is used with the division sign. The easiest way to read them is to perform the division operation mentally and then read thus:— "ten minus four equals six," "ten minus three equals seven," and so on.

The exercises on **page 66,** using the signs plus and minus, should be read quickly giving results only thus:

7, 9, 10, 4, 2, 7.   3, 8, 6, 10, 3, 2.
9, 3, 5, 4, 9, 7.   Etc.

HAND-BOOK

TO

PUBLIC SCHOOL ARITHMETIC.

GRADE II.

Before the introduction of the number eleven, a review may be given on the writing and recognition of numbers containing tens and units using bundles of tens and single ones.

The teacher holding up two bundles and one single one asks class to indicate on the board how many she has. Pupils write 21. Continue using a great variety of numbers until pupils can do this.

The teacher writes on board 24, 13, 17, 34, 15, 11, 30, etc., and asks class to show with bundles and single ones what she has written in each case. This will fix in their minds the position of the tens and the units and their value.

The class is now ready for the study of the number eleven. In the above work they will have learned that eleven is made up of one bundle of ten and one single one, that the one on the left is the one bundle of ten and the one on the right is the single one.

Using the bundle of ten (not broken up), give a rapid review of the number ten. The class will

now be able to give all the facts of eleven, in subtraction, addition, quotition and multiplication, deducing these from their knowledge of the number ten.

Picture numbers should be discontinued after the number ten.

$11 = 10 + 1$	$10 - 3 = 7$	$11 = 10 + 1$
$10 - 8 = 2$	$11 - 3 = 7 + 1 = 8$	$11 = \quad (2) +$
$11 - 8 = 3$	$8 + 3 = 11$	
$3 + 8 = 11$	$8 = 2\ (3) + 2$	$10 - 2 =$
	$11 = 3\ (3) + 2$	$11 - 2 =$
	$10 - 5 = 5$	$9 + 2 =$
	$11 - 5 = 5 + 1 = 6$	
	$6 + 5 = 11$	
	$11 = 2\ (5) + 1$	

To test their knowledge they may be asked to write the addition facts of eleven on the board, the teacher giving them the type she wishes used, thus $11 = 5 + 6$ or $5 + 6 = 11$. If the subtraction facts are asked for the type should also be given, thus $11 - 5 = 6$ or $5 = 11 - 6$. By having all the pupils working at the board these and similar exercises may be used as tests in speed as well as in accuracy.

The work of interpreting these little exercises in the form of problems should be continued throughout the course.

The advantages of this will have been seen by the teacher long before the pupils have reached this stage.

Problems are introduced in Grade One, but the work should be oral, both in that grade and in Grade Two until they know the numbers up to and including twenty, thoroughly. The written work will be dealt with in connection with section 130 and the following sections.

It is advisable to have the pupils express the thoughts in the problems in well worded sentences. Every problem should be analysed carefully to see what is given and what is required.

Page 68.—Section 5. Example 1. Ask different pupils to read the problem until all are perfectly familiar with it.

TEACHER.—"What are we told?" (Allow only one statement to be given at a time).

PUPIL.—"Fred had 11 pencils."

"He lost one pencil."

"He gave his sister one-half of what he had left."

TEACHER.—"What are we asked to find?"

PUPIL.—"How many pencils his sister got."

The pupil may now word the solution something like this :—

Fred had 11 pencils.

He lost 1 pencil.

He had 10 pencils left.

He gave his sister one-half of 10 pencils.

He gave his sister 5 pencils.

Example 2.—Jennie had 11 cents. She spent 2 cents for candy. She had 9 cents left. She divided 9 cents equally among 3 girls. Each girl got 3 cents.

Section 8.—To read exercises similar to these it is well to give results only, thus :

11, 5, 7. 11, 3, 10. 10, 6, 11, etc.

The number twelve may be taught in the same way as the number eleven deducing the facts from their knowledge of the number ten.

In 12, what does the 2 mean? What does the 1 mean?

12 = 2 +	12 = 10 +	12 − 1 =
12 = 10 +	10 − 7 =	11 + 1 =
12 = (10) +	12 − 7 = 3 +	12 − 5 =
12 − 10 =	12 − 7 =	7 + 5 =
2 + 10 =	5 + 7 =	12 = (5) +
12 − 2 =	10 − 4 =	10 − 3 =
10 + 2 =	12 − 4 = 6 +	12 − 3 = 7 +

12 = (2)	12 − 4 =	12 − 3 =
6 *t* 2 =	8 + 4 =	9 + 3 =
one-sixth of 12 =	12 = (4)	12 = (3)
	3 *t* 4 =	4 *t* 3 =
	one-third of 12 =	one-fourth of 12 =
12 = 10 +	12 = 10 +	12 = 10 +
10 − 6 =	10 − 9 =	10 − 8 =
12 − 6 = 4 +	12 − 9 =	12 − 8 =
12 − 6 =	3 + 9 =	4 + 8 =
6 + 6 =		
12 = (6)		
2 *t* 6 =		
one-half of 12 =		

Page 70 is made up of work upon the number twelve, but, as with all preceding work, the pupils should not be asked to do this until the number has been thoroughly mastered in class.

The arrangement suggested in Section 20, may be used very early in teaching the number twelve. This will assist in obtaining the quotition and multiplication facts.

On **page 71,** Section 19, the terms "foot" and "inch" are introduced.

Many methods suggest themselves to the teacher. One which may be helpful is to give the pupils rulers, scissors and paper. Ask them to cut several strips of paper a foot long and an

inch wide. To see the relation of one inch to one foot, fold the strip so that the ends will be exactly together and crease. Pupils know that the paper as folded shows one-half a foot. Fold the folded paper so that the ends are together and crease, now one-fourth is seen.

Fold the fourth into three equal parts and crease. Open the paper and count the parts. This must be done several times before the pupils will be able to do it well.

Using the same strip, they may be asked to show one-half a foot, one-fourth, and one-third by folding each time. The number of inches in each may be given as the paper is folded.

Before making use of the questions in Section 20 one-sixth should be taught. It may be taught in the same way as one-fourth, using strips of paper. Fold the strip as for one-half, crease and without opening, fold into three equal parts. When open they will see that there are six equal parts. The teacher tells them that each part is called one-sixth. By folding again to show one-half they will see that the half is divided into three equal parts, and that therefore one-sixth is the same as one-third of one-half. The pupils will see that and should not be told. They can also tell how they obtained the one-sixth, viz.,

first fold the paper so as to get one-half, then fold the half so as to get one-third of it. Later the paper may be cut or torn at the crease at each step. Then the operation may be described in this way. First, divide it into two equal parts; then divide each half into three equal parts.

Again, take another slip of paper and fold it so as to show one-third. Without opening fold again, as for one-half. The pupils will see that this method of folding will also give one-sixth, and that one-sixth is one-half of one-third. As before, they will state how they obtained this. They will be able to tell how many sixths in one-half, one third, two-thirds, the difference between one-half and one-third, etc.

This may be followed by a further study of the arrangement of 12 as indicated in Section 20.

This may be presented and developed in class thus,—The teacher puts down 12 and asks, "What two equal numbers make 12?" Making the bracket as indicated, place the two 6's.

Bracketing each 6 as indicated, she asks, "What three equal numbers make 6?" Place the three 2's. "What three equal numbers make 6?" Place another three 2's. Bracketing two of the 2's, she asks, "what do two 2's make?"

Place the 4's as indicated. The pupils see at a glance two 6's make 12, six 2's make 12 and three 4's make 12; that one-half of 12 is 6, one-third of 12 is 4, one-sixth of 12 is 2. The analysis at the right will show that four 3's make 12 and that one-fourth of 12 is 3.

To test the class further ask them to write the multiplication and quotition facts on board, each time indicating which type is wanted, thus:

$$
\begin{array}{l}
12 = 2\ t\ 6 \text{ or } 2\ t\ 6 = 12 \\
12 = 6\ t\ 2 \text{ or } 6\ t\ 2 = 12 \\
12 = 3\ t\ 4 \text{ or } 3\ t\ 4 = 12 \\
12 = 4\ t\ 3 \text{ or } 4\ t\ 3 = 12 \\
12 = 12\ t\ 1 \text{ or } 12\ t\ 1 = 12 \\
12 \div 6 = 2 \\
12 \div 2 = 6 \\
12 \div 4 = 3 \\
12 \div 3 = 4 \\
12 \div 12 = 1 \\
12 \div 1 = 12
\end{array}
$$

The following is an exercise which will serve as a review and test of previous numbers. This might be used with success with any number after the number nine has been taught.

The class at the board. Ask them to imagine they have twelve rabbits, apples, or oranges. Tell them to give them all away, giving some to

one boy and some to another, and write their problems. They make their problems mentally and write, thus:

$$12 - 3 - 9 = 0$$
$$12 - 7 - 5 = 0$$
$$12 - 10 - 2 = 0$$
$$12 - 8 - 4 = 0, \text{ etc.}$$

Occasionally ask for an interpretation of what they have written.

Again, they may be asked to give them all away but one. They will at once see that they must give away eleven. This will be a review and test of their knowledge of the addition facts of eleven, thus:

$$12 - 4 - 7 = 1$$
$$12 - 6 - 5 = 1$$
$$12 - 1 - 10 = 1, \text{ etc.}$$

Again, they may be asked to give them all away but two, three, etc.

Another helpful exercise is to ask them to put three numbers together to make twelve, using a different group each time. Here again they should frame problems mentally, and occasionally interpret aloud what they have written.

I had 3 apples. My sister gave me 4 and I bought 5. Now I have 12 apples.

$$3 + 4 + 5 = 12$$
$$2 + 1 + 9 = 12$$
$$5 + 3 + 4 = 12, \text{ etc.}$$

Too much emphasis cannot be laid on drill. Thoroughness is the key to success in this as in all subjects.

All the facts of one number are very easily learned. It is only when there are exercises where different numbers are involved, that there is a test of their knowledge. So there should be constant drill and review, both oral and written.

From the pupils' knowledge of one-half, one-third, one-fourth, one-fifth, one-sixth (and how obtained), they will very readily understand that if anything is divided into seven equal parts, each part is called one-seventh.

Section 43 will suggest a method of teaching one-seventh.

The answers to the questions in *Sections 54, 71, 87* and *97* may be written thus:

$15 = 15\ t\ 1$	or thus,	$15 = 15\ (1)$
$15 = 7\ t\ 2 + 1$		$15 = 7\ (2) + 1$
$15 = 5\ t\ 3$		$15 = 5\ (3)$
$15 = 3\ t\ 4 + 3$		$15 = 3\ (4) + 3$
$15 = 3\ t\ 5$		$15 = 3\ (5)$
$15 = 2\ t\ 6 + 3$, etc.		$15 = 2\ (6) + 3$

Here is a simple device that may be used in teaching the facts of the numbers fifteen to twenty. They know that fifteen is made up of ten and five. Obtain the addition facts of 5 and write on the board thus:

$$\begin{array}{cccc} 4 & 3 & 1 & 2 \\ 1 & 2 & 4 & 3 \\ \hline 5 & 5 & 5 & 5 \end{array}$$

Lead the class to see that to make 15 out of the above 5's, we must add 10. Place the 1 to the left of the 5 in the first example. They will see that we must put a ten with either the 4 or the 1 in order to make 15. If the ten is added to the 4 it is well to add it to the top line all the way through.

It will then read:

$$\begin{array}{rrrr} 14 & 13 & 11 & 12 \\ 1 & 2 & 4 & 3 \\ \hline 15 & 15 & 15 & 15 \end{array}$$

They know that 10 and 5 make 15, so that can be added to the line. Since 10 and 5 make 15, then 9 and 6 make 15. Since 7 and 7 make 14, then 8 and 7 make 15; or since 10 and 5 make 15, then 8 and 7 make 15.

Then the line will read:

14	13	11	12	10	9	8
1	2	4	3	5	6	7
—	—	—	—	—	—	—
15	15	15	15	15	15	15

The pupils will readily see that there are only two addition facts that are new to them, 9 and 6, and 8 and 7.

The facts of 16 will appear thus, not necessarily in this order, nor in any particular order.

13	14	11	12	15	10	9	8
3	2	5	4	1	6	7	8
—	—	—	—	—	—	—	—
16	16	16	16	16	16	16	16

Here the only new facts are 9 and 7 and 8 and 8, but the latter one presents no difficulty.

The higher the number is, the fewer are the new addition and subtraction facts.

From the above the quotition and multiplication facts are easily seen. Since 14 and 2 make 16, then two 7's and 2 make 16. Since 12 and 4 make 16, then two 6's and 4 make 16. Since 15 and 1 make 16, then three 5's and 1 make 16, etc.

After all the numbers up to and including 19 are taught, special drill may be given on adding 9 to any number.

If

10	10	10	10	10	10	10	10
9	1	8	6	4	7	2	5
—	—	—	—	—	—	—	—
19	11	18	16	14	17	12	15

Then

9	9	9	9	9	9	9	9
9	1	8	6	4	7	2	5
—	—	—	—	—	—	—	—
18	10	17	15	13	16	11	14

Page 85.—Section 64. One-eighth may be taught by using the strips of paper. Fold, showing one-half, fold again showing one-fourth. While folded, fold again as for one-half and crease. Open. If necessary the pupils count the equal parts and find there are eight. If they do not know that each part is called one-eighth, tell them.

This should be done again and again until every pupil sees that one-eighth is one-half of one-fourth, and one-fourth of one-half. A pupil should not be told what he can find out for himself.

Section 65.—The simplest way to read this is to give results only, thus: "four plus 5 equals 9," "seven plus three equals ten."

Page 87.—Section 71 may be written thus:

16 = 16 *t* 1	16 = 2 *t* 6 + 4
16 = 8 *t* 2	16 = 2 *t* 7 + 2
16 = 5 *t* 3 + 1	16 = 2 *t* 8
16 = 4 *t* 4	16 = 1 *t* 9 + 7
16 = 3 *t* 5 + 1	16 = 1 *t* 10 + 6, etc.

Or thus:

16 = 16 (1)	16 = 1 (9) + 7
16 = 8 (2)	16 = 1 (10) + 6
16 = 5 (3) + 1	16 = 1 (11) + 5
16 = 4 (4)	16 = 1 (12) + 4
16 = 3 (5) + 1	16 = 1 (13) + 3
16 = 2 (6) + 4	16 = 1 (14) + 2
16 = 2 (7) + 2	16 = 1 (15) + 1
16 = 2 (8)	16 = 1 (16)

Page 107.—Section 130. As the pupils now have a thorough knowledge of the numbers 1 to 20, and they have solved problems orally, they are ready to write the solutions.

To teach a problem it is necessary that every pupil knows what it contains. To this end several pupils should be asked to read it aloud. Then proceed as outlined on preceding pages. Section 130. Example 1.

TEACHER.—"What are we told in this problem?" Require only one statement or fact in each answer.

Pupils.—"A boy had 20 cents." "He spent 5 cents for candy."

Teacher.—"What are we asked to find?"

Pupil.—"How much he had left."

Teacher.—"What will be the name of the answer?"

Pupil.—"Cents."

The solution should be written in full sentences, attention being paid to capitals and punctuation.

A boy had 20 cents.

He spent 5 cents.

He had left 20 cents − 5 cents = 15 cents.

Example 2.

A boy had 19 cents.

He spent 6 cents + 7 cents = 13 cents.

He had left 19 cents − 13 cents = 6 cents.

Example 3.

Jim spent 8 cents.

He had left 10 cents.

He had at first 8 cents + 10 cents = 18 cents.

Example 4.

Henry caught 9 fish.

Jack caught 7 fish.

Both caught 9 fish + 7 fish = 16 fish.

For further exercises, the problems on preceding pages may be used, care being taken to select only those involving addition and subtraction at this stage.

Page 107.—Section 132. It is wise to analyse several problems before attempting solutions. The teacher will find it very profitable to write out the analysis of each problem in some such way as this. If this is done, the pupils will have no difficulty. They will learn to reason for themselves and not have to ask, "shall I multiply or divide, add or subtract?"

Section 132.—Example 1.

Analysis:

TEACHER.—"What are we told?"

PUPIL.—"One spool is worth 5 cents." "The number bought."

TEACHER.—"What are we asked to find?"

PUPIL.—"What 2 spools would cost."

$$\text{Cost of all}\begin{cases}\text{Cost of 1.}\\ \\ \text{Number bought.}\end{cases}$$

Solution:

Repeat the above questions.

TEACHER.—"What are we asked to find?"

PUPIL.—"What 2 spools would cost."

TEACHER.—"What will be the name of the answer?"

PUPIL.—"Cents."

TEACHER.—"We always write our statements so as to have the answer on the right side of our paper."

TEACHER.—"What are we told?"

PUPIL.—"The cost of one spool." "The number spools bought."

TEACHER.—"What is the cost of one spool?"

PUPIL.—"One spool costs 5 cents."

Teacher writes this on board.

Question so as to lead the pupils to see that that statement is written correctly.

TEACHER.—"What is to be the name of the answer?"

PUPIL.—"Cents."

TEACHER.—"Where should that appear?"

PUPIL.—"On the right hand side."

TEACHER.—"Read what is written." "Where is the word cents?"

PUPIL.—"On the right hand side."

TEACHER.—"What are we asked to find?"

PUPIL.—"The cost of two spools."

TEACHER.—"If one spool cost 5 cents what will 2 spools cost?"

PUPIL.—"Ten cents."

TEACHER.—"How did you get it?"

PUPIL.—"Two spools will cost twice as much as one spool."

Teacher writes:

1 spool cost 5 cents.

2 spools cost 2 times 5 cents = 10 cents.

Example 2.

Analysis:

$$\text{Cost of 3 oranges}\begin{cases}\text{Number bought, 3.}\\ \text{Cost of 1, 5 cents.}\end{cases}$$

Solution:

1 orange costs 5 cents.

3 oranges cost 3 times 5 cents = 15 cents.

Example 4.

In 1 week there are 7 days.

In 2 weeks there are 2 times 7 days = 14 days.

Page 108.—Section 134 is a combination of multiplication and addition or subtraction.

Example 1.

Analysis:

$$\text{What he had left}\begin{cases}\text{What he had at first, 20 cents.}\\ \text{What he spent}\begin{cases}\text{Number of oranges, 3.}\\ \text{Cost of 1 orange, 4 cts.}\end{cases}\end{cases}$$

Class read problem several times.

TEACHER.—"What are we asked to find?"

PUPIL.—"How much Jack had left."

Teacher writes this. (See Analysis).

TEACHER.—"What must we know in order to find what he had left?"

PUPIL.—"What he had at first and what he spent."

Teacher writes this. (See Analysis).

TEACHER.—"Are we told what he had at first?"

PUPIL.—"Yes, 20 cents."

Teacher writes "20 cents."

TEACHER.—"Then we do not have to find that." "Are we told what he spent altogether?"

PUPIL.—"No."

TEACHER.—"Then we must find it."

Teacher places bracket. (See Analysis).

TEACHER.—"How many things must we know to find what he spent altogether?"

PUPIL.—"Two."

TEACHER.—"What are they?"

PUPIL.—"The cost of one and the number bought."

Teacher writes. (See Analysis).

TEACHER.—"Are we told the cost of one?"

PUPIL.—"Yes, 4 cents."

Teacher writes.

TEACHER.—"Are we told the number bought?"

PUPIL.—"Yes, 3."

Teacher writes.

TEACHER.—"Now we know everything we need to find what Jack had left." "What shall we find first?"

The pupils will see that they must begin where they have two things they know. Therefore, just find what he spent. As they know how to do this, they can give the statements at once.

Teacher writes as pupils dictate,

1 orange costs 4 cents.

3 oranges cost 3 times 4 cents = 12 cents.

TEACHER.—"What have we found?"

PUPIL.—"What he spent."

Teacher writes "12 cents" in analysis beside the words "what he spent," and erases "number of oranges" and "cost of one," as they are not needed any longer.

Teacher.—"What do we know now?"

Pupil.—"What Jack had at first and what he spent."

Teacher.—"What can we find?"

Pupil.—"What he had left."

Teacher.—"What had he left?"

Pupil.—"He had left 20 cents − 12 cents = 8 cents.

Teacher writes this with the first part of the solution.

1 orange costs 4 cents.

3 oranges cost 3 times 4 cents = 12 cents.

He had left 20 cents − 12 cents = 8 cents.

Example 2.

Analysis:

$$\text{What she spent}\begin{cases}\text{What she spent for candy, 9 cents.}\\ \text{What she spent for pop-corn}\begin{cases}\text{Cost of 1 bag, 5 cts.}\\ \text{Number of bags, 2.}\end{cases}\end{cases}$$

Example 2.

Solution:

1 bag of pop-corn cost 5 cents.

2 bags of pop-corn cost 2 times 5 cents = 10 cents.

She spent altogether 9 cents + 10 cents = 19 cents.

Example 3.

Analysis:

$$\text{How much he had left}\begin{cases}\text{What he had at first}\begin{cases}\text{What he earned in 1 day, \$3.}\\ \text{Number of days he worked, 6.}\end{cases}\\ \text{What he spent}\begin{cases}\text{Hat, \$2.}\\ \text{Shoes, \$5.}\\ \text{Coat, \$4.}\end{cases}\end{cases}$$

Example 3.

Solution:

In 1 day he earned 3 dollars.

In 6 days he earned 6 times 3 dollars = 18 dollars.

He spent 2 dollars + 5 dollars + 4 dollars = 11 dollars.

He had left 18 dollars − 11 dollars = 7 dollars.

Page 109.—Section 135. This should be read giving results only, thus:

20, 14, 8, 13. 7, 19, 13, 9. 13, 7, 15, 6, 20.
20, 13, 7, 16, 4. 4, 13, 19, 8.

Section 137.—First column may be read:

20 equals 18 plus 2.
18 equals 2 times 9.
20 equals 12 plus 8.
12 equals 4 times 3.
18 equals 14 plus 4.
14 equals 7 times 2, etc.

Section 137.—Second column may be read:

18 equals 5 plus 13.
15 divided by 3 equals 5.
19 equals 7 plus 12.
14 divided by 2 equals 7.

Section 140 introduces problems in partition.

Example 1.

Analysis:

Have the problem read several times.

TEACHER.—"What are we asked to find?"

PUPIL.—"The value or cost of 1 apple."

Teacher writes as indicated below.

TEACHER.—"How many things must we know?"

PUPIL.—"Two."

TEACHER.—"What are they?"

PUPIL.—"The cost of all the apples and the number bought."

Teacher writes.

TEACHER.—"What is the cost of all?"

PUPIL.—"Six cents."

TEACHER.—"How many apples were bought?"

PUPIL.—"Two."

$$\text{Cost of 1 apple}\left\{\begin{array}{l}\text{Cost of all, 6 cents.}\\ \text{Number bought, 2 apples.}\end{array}\right.$$

Solution:

TEACHER.—"What is the name of the answer?"

PUPIL.—"Cents."

TEACHER.—"On which side will the answer appear?"

PUPIL.—"The right side."

TEACHER.—"What do we know?"

PUPIL.—"The cost of two apples."

TEACHER.—"How shall I write it?"

PUPIL.—"2 apples cost 6 cents."

TEACHER.—"What are we asked to find?"

PUPIL.—"The cost of one."

TEACHER.—"If two apples cost 6 cents, what will one cost?"

PUPIL.—"One will cost just one-half as much as two."

Teacher writes:

2 apples costs 6 cents.

1 apple costs one-half of 6 cents = 3 cents.

Example 2.

Analysis:

$$\text{What did one boy earn}\left\{\begin{array}{l}\text{What was earned by all, 20 cents.}\\ \text{How many boys there were, 2.}\end{array}\right.$$

Solution:

2 boys earned 20 cents.

1 boy earned one-half of 20 cents = 10 cents.

Example 3.—Analysis:

$$\text{The number of oranges one child got}\left\{\begin{array}{l}\text{The number all had, 15 oranges.}\\ \text{The number of children, 5.}\end{array}\right.$$

Solution:

5 children received 15 oranges.

1 child received one-fifth of 15 oranges = 3 oranges.

Page 112.—Section 144. This section contains problems involving the two preceding types combined, viz., partition and multiplication.

Example 3.

Solution:

3 oranges cost 12 cents.

1 orange costs one-third of 12 cents = 4 cents.

5 oranges cost 5 times 4 cents = 20 cents.

Page 114.—Section 150. In this section, quotition problems are introduced.

Analysis:

$$\text{The number bought} \begin{cases} \text{Cost of 1, 5 cents.} \\ \text{Cost of all, 15 cents.} \end{cases}$$

Solution:

5 cents buys 1 orange.

15 cents buys as many oranges as the number of times 5 cents is contained in 15 cents.

5 cents is contained in 15 cents, 3 times.

15 cents buys 3 oranges.

This method of writing the solution should be continued until the pupils are perfectly familiar with the reasoning. Then the written work may be shortened, but the oral work given in full.

5 cents buys 1 orange.

15 cents buys (15 ÷ 5) oranges = 3 oranges.

Example 2.

Analysis:

$$\text{The number of tons bought}\left\{\begin{array}{l}\text{Cost of one ton, 6 dollars.}\\ \text{Cost of all, 18 dollars.}\end{array}\right.$$

Solution:

6 dollars buys 1 ton.

18 dollars buys as many tons as the number of times that 6 dollars is contained in 18 dollars.

6 dollars is contained in 18 dollars, 3 times.

18 dollars buys 3 tons.

6 dollars buys 1 ton.

18 dollars buys $(18 \div 6)$ tons $= 3$ tons.

Example 3.

Analysis:

$$\text{The number of quarts picked}\left\{\begin{array}{l}\text{Number of cents received for picking 1 quart.}\\ \text{Number of cents received for picking all.}\end{array}\right.$$

Solution:

2 cents are received for picking one quart.

12 cents are received for picking as many quarts as there are 2's in 12.

There are six 2's in 12.

12 cents buys 6 quarts.

2 cents are received for picking 1 quart.

12 cents are received for picking $(12 \div 2)$ quarts $=$ 6 quarts.

Page 115.—Section 153 may be read :—"Four times two equals eight." "Nine times two equals eighteen." "Three times two equals six," and so on, or thus :—"Two times four equals eight." "Three times six equals eighteen." "Three times two equals six," and so on.

Page 116.—Section 154. Example 1.

2 boys dig a garden in 3 days.

1 boy digs it in 2 times 3 days $=$ 6 days.

Page 116.—Section 156. Example 1.

Analysis:

The number of cents left { Number of cents she had at first, 19 cents.
Number of cents spent, 5 cents, 8 cents, 2 cents.

Solution:

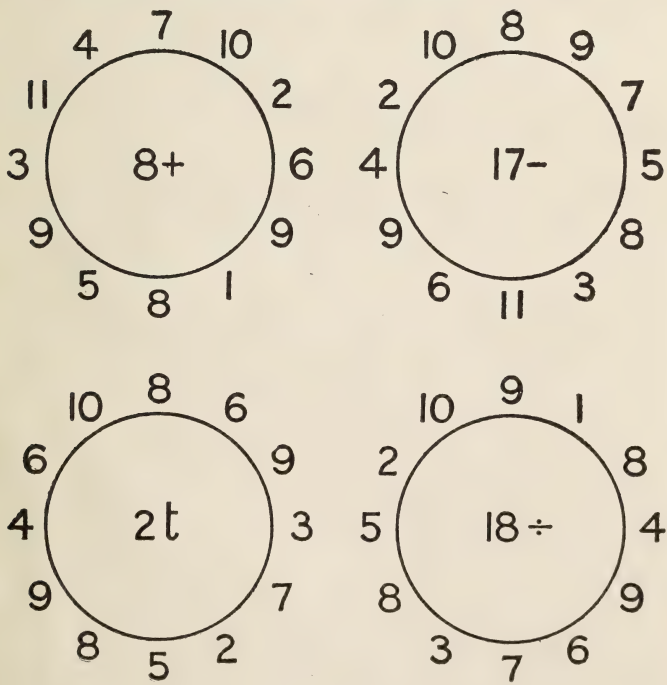
Jennie spent 5 cents $+$ 8 cents $+$ 2 cents $=$ 15 cents.

She had left 19 cents $-$ 15 cents $=$ 4 cents.

Below is given a few of the methods which have proven helpful in rapid drill.

Place figures in a row on the board. The teacher points to those she wishes the pupils to add. The figures may also be placed in any other position. This may be made a drill in addition, subtraction, multiplication or quotition.

Draw a circle on the board and around it place figures, and a figure in the centre with a sign, showing the operation you wish performed, thus:



For drill in addition, a square with diagonals, may be used. Place figures in any place desired and point to those you wish the class to add.

The teacher calls out a number, say twelve, a pupil gives two numbers when added together that make twelve, or two numbers multiplied together that make twelve, or two of which the difference is twelve as the teacher requests.

If a small number be given, it might be used as a drill in partition. Suppose the number "four" be given. The answer will be, "One-half of eight is four," or "one-third of twelve is four," or "one-fourth of sixteen is four," or "one-fifth of twenty is four."

Place a row of figures on the board, have a pupil put a figure underneath each so that when the two are added, the sum will be a given number. For this, two pupils may go to the board at once, one beginning at one end and the other, at the other end. The pupils delight in seeing who can get the greater number done.

Place columns of figures, two figures in each column. Ask the pupils to place another figure in each column so that the sum will be a given number.